

Übungen zu **Numerik (für Geowissenschaftler)**

Blatt 4

Ü1 (*Condition number*) Let $A = \begin{bmatrix} 2 & 0 \\ 0 & -5 \end{bmatrix}$.

What is the condition number $\text{cond}(A) = \|A\| \cdot \|A^{-1}\|$ of A with regard to the 1-, 2- and ∞ -Norm.

Ü2 (*Diagonal numbering*)

Row-wise numbering of the unknowns, i.e.

$u = (u_{1,1}, u_{2,1}, u_{3,1}, u_{1,2}, u_{2,2}, u_{3,2}, u_{1,3}, u_{2,3}, u_{3,3})^T$ gives the linear system

$$\begin{bmatrix} 4 & -1 & 0 & -1 & 0 & 0 & 0 & 0 & 0 \\ -1 & 4 & -1 & 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & -1 & 4 & 0 & 0 & -1 & 0 & 0 & 0 \\ -1 & 0 & 0 & 4 & -1 & 0 & -1 & 0 & 0 \\ 0 & -1 & 0 & -1 & 4 & -1 & 0 & -1 & 0 \\ 0 & 0 & -1 & 0 & -1 & 4 & 0 & 0 & -1 \\ 0 & 0 & 0 & -1 & 0 & 0 & 4 & -1 & 0 \\ 0 & 0 & 0 & 0 & -1 & 0 & -1 & 4 & -1 \\ 0 & 0 & 0 & 0 & 0 & -1 & 0 & -1 & 4 \end{bmatrix} \begin{pmatrix} u_{1,1} \\ u_{2,1} \\ u_{3,1} \\ u_{1,2} \\ u_{2,2} \\ u_{3,2} \\ u_{1,3} \\ u_{2,3} \\ u_{3,3} \end{pmatrix} = f$$

for solving Laplace's equation numerically. (c.f. Today's exercises)

How does the matrix look like, if we use the numbering scheme

$u = (u_{1,1}, u_{1,2}, u_{2,1}, u_{1,3}, u_{2,2}, u_{3,1}, u_{2,3}, u_{3,2}, u_{3,3})^T$ for the unknowns?

Ü3 (*Rotation*) Let $A_\alpha = \begin{bmatrix} \cos \alpha & -\sin \alpha & 0 \\ \sin \alpha & \cos \alpha & 0 \\ 0 & 0 & 1 \end{bmatrix}$

a) What are A_π (i.e. $\alpha = \pi$), $A_{\frac{\pi}{2}}$, $A_{-\frac{\pi}{2}}$, and $A_{2\pi}$?

b) Given $\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} \in \mathbf{R}^3$: What is Ax compared to x ?

c) Calculate $A_\alpha^2 = A_\alpha \cdot A_\alpha$. Comments?

d) What are the eigenvalues of A_α ?